

Instructions: Give complete solutions to the following problems be sure to provide all the necessary steps to support your answers.

1. Find the characteristic equation for the matrix A and determine if the solutions are real or complex, and the multiplicity of each.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ 2 & 3 & 2 \end{bmatrix}$$

2. Find the eigenvalues and the eigenvectors of the matrix A. then determine both the algebraic and geometric multiplicities of each eigenvalue.

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

3. Find the eigenvalues and the eigenvectors of the matrix A , then determine both the algebraic and geometric multiplicities of each eigenvalue, and verify that

$$\text{Trace} = \lambda_1 + \lambda_2 + \lambda_3, \text{ and } \text{Det}(\mathbf{A}) = \lambda_1 \lambda_2 \lambda_3$$

$$\mathbf{A} = \begin{bmatrix} 2 & 3 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix}$$

4. Find two matrices P that diagonalize \mathbf{A} , and in each case determine $\mathbf{P}^{-1}\mathbf{A}\mathbf{P}$

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

5. Solve the given system of linear equations initial value problem.

$$\begin{cases} \frac{dy_1}{dt} = -0.2y_1 \\ \frac{dy_2}{dt} = 0.2y_1 - 0.1y_2, & y_1(0) = 1, y_2(0) = 1, y_3(0) = 1, \\ \frac{dy_3}{dt} = 0.1y_3 \end{cases}$$