

Instructions: Write complete legible solutions to the following problems in the space provided. Be sure to supply all the necessary steps that lead to your answers.

1. Find the characteristic equation for the matrix A and determine if the solutions are real or complex, and the multiplicity of each.

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

2. Find the eigenvalues and the eigenvectors of the given matrices. then determine both the algebraic and geometric multiplicities of each eigenvalue.

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

3. Find the eigenvalues and the eigenvectors of the matrix \mathbf{A} , then determine both the algebraic and geometric multiplicities of each eigenvalue, and verify that

$$\text{Trace} = \lambda_1 + \lambda_2 + \lambda_3, \text{ and } \text{Det}(\mathbf{A}) = \lambda_1 \lambda_2 \lambda_3$$

$$\mathbf{A} = \begin{bmatrix} 2 & 3 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix}$$

4. Find a matrix \mathbf{P} that diagonalize \mathbf{A} , and determine $\mathbf{P}^{-1}\mathbf{A}\mathbf{P}$

$$\mathbf{A} = \begin{bmatrix} 3 & -1 & -1 \\ -12 & 0 & 5 \\ 4 & -2 & -1 \end{bmatrix}$$