

Instructions: Write complete legible solutions to the following problems in the space provided. Be sure to supply all the necessary steps that lead to your answers.

1. For the system  $Ax=b$  below, find conditions on  $b$  so the given system is consistent

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 5 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

2. Use Gaussian eliminations to produce the inverse of the coefficient matrix then use it to find solutions the system for the given input matrices  $b_i$ .

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = b, \quad b_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix}, \quad b_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

3. Use LU factorization and forward and backward substitution to solve the given system

$$\begin{cases} x + 2y = 3 \\ x + y = 2 \\ x + z = 2 \end{cases}$$