Give complete solutions to the following problems. Be sure to provide all the necessary steps to support your answers.

1. Show that the set of all 2x2 Diagonal matrices form an Abelian group under the usual matrix addition.

2. Determine if the set of all ordered pairs of the form $(1, x)$ with the operations
$$(1, y) + (1, z) = (1, y + z), \text{ and } k(1, y) = (1, ky)$$
Defines a vector space.

3. Determine which of the following form a subspace of $R^3$

   a. All vectors of the form $(a, b, c)$, where $c = a + b + 1$.

   b. All vectors of the form $(a, b, c)$, where $c = a + b$.

4. Le $S = \{(1,-2,1),(1,1,2),(3,-3,4)\}$, determine which of the following vectors is a member of Span(S) ,

a. $(1,-8,-1)$ b. $(5,1,8)$ c. $(1,2,1)$

5. Determine if the given vectors span the vector space of polynomial of degree three or less.

$\mathbf{p}_1 = 2 + 2x + 2x^2,\ \mathbf{p}_1 = 3x^2, \mathbf{p}_1 = x + x^2,$

6. Determine if the set of vectors are linear independent $1,\ \sin x,\ \sin 2x$

5. Express the vector $6 + 11x + 6x^2$ as linear combination of

$\mathbf{P}_1 = 2 + x + 4x^2, \mathbf{P}_1 = 1 - x + 3x^2, \mathbf{P}_1 = 3 + 2x + 5x^2,$