Instructions: Write complete legible solutions to the following problems in the space provided. Be sure to supply all the necessary steps that lead to your answers.

1. Let $S = \{(1,-2,1),(1,0,2),(1,1,2),(1,1,1)\}$

a. Find the dimension the vector space V= Span (S)

b. Give a basis for V.

2. Find a basis for the solution set of the given system.

$$x_1 + x_2 - x_3 + x_4 = 0$$
$$x_1 - 2x_2 + x_3 + x_4 = 0$$
$$x_1 + 2x_2 - x_3 - 2x_4 = 0$$

3. Consider the bases

$$B=\{u_1,u_2,u_3\},\quad B'=\{u_1',u_2',u_3'\} \text{ for } \mathrm{R}^3, \text{ where}$$

$$u_1=\begin{bmatrix}1\\2\\1\end{bmatrix},u_2=\begin{bmatrix}3\\1\\1\end{bmatrix},u_3=\begin{bmatrix}1\\-1\\1\end{bmatrix},\ u_1'=\begin{bmatrix}1\\3\\1\end{bmatrix},u_2'=\begin{bmatrix}2\\1\\1\end{bmatrix}u_3'=\begin{bmatrix}-1\\1\\1\end{bmatrix}$$

a. Find the transition matrix of from B to B', $\mathrm{P}_{\mathrm{B}\to\mathrm{B}'}$

b. Compute the coordinate vector of w relative to B, $[w]_B$,where $w=\begin{bmatrix}1\\3\\5\end{bmatrix}$

c. Compute the coordinate vector of w relative to, B', $[w]_{B'}$, **using the transition matrix in part a.**

d. Suppose there is an object located at the point (x_1,x_2,x_3), a dispatcher gave a coordinate of the object relative to the basis B of $(-2,1,3)$ find the actual location of the object. What is the transition matrix in this case?