
Instructions: Write complete legible solutions to the following problems in the space provided. Be sure to supply all the necessary steps that lead to your answers.

1. Use vector projection to find the closest vector to \mathbf{u} on the space \mathbf{W} spanned by the set S , then use the projection to find the distance from \mathbf{u} to the space \mathbf{W} , the error vector in the approximation. Use a picture of \mathbf{W} and \mathbf{u} to illustrate.

$$\mathbf{u} = (1, 2, 1), \quad S = \{(1, -1, 1), (1, 1, 2)\}$$

2. Find the standard matrix for the orthogonal projection onto the space \mathbf{W} spanned by the set S , then use the matrix to find the projection of the vector \mathbf{u} onto \mathbf{W} .

$$S = \{(2, 1, 1, 1), (1, 0, 1, 1), (-2, -2, 0, -1)\}, \quad \mathbf{u} = (2, 3, 9, 6).$$

3. Let $S = \{(10, -5, 5), (0, 1, 2), (3, 6, 2)\}$ and $\mathbf{W} = \text{Span}(S)$
- a. Use the Gram-Schmidt process to find an orthonormal basis for \mathbf{W} .
- b. Use the Euclidean inner product on \mathbb{R}^3 to find the coordinates of \mathbf{u} relative to the ONB of \mathbf{W} , where $\mathbf{u} = (3, 2, 1)$.

4. Let $S = \{(0, 1, 2, 1), (0, 1, 3, 1), (1, 1, 1, 0)\}$, and $\mathbf{W} = \text{Span}(S)$
- a. Use the Gram-Schmidt process to find an orthonormal basis for \mathbf{W} .
- b. Use the Euclidean inner product on \mathbb{R}^4 to find the orthogonal projection of \mathbf{u} onto the ONB of \mathbf{W} , where $\mathbf{u} = (4, 3, 2, 1)$.

- c. Find the standard matrix for the projection of \mathbf{u} onto the ONB of \mathbf{W} and use it to confirm your answer in part a.