

Instructions: Write complete legible solutions to the following problems in the space provided. Be sure to supply all the necessary steps that lead to your answers.

1. Solve the given system of linear equations initial value problem by converting the system into pure exponential system of equations.

$$\begin{cases} \frac{dy_1}{dt} = -0.2y_1 \\ \frac{dy_2}{dt} = 0.2y_1 - 0.1y_2, \\ \frac{dy_3}{dt} = 0.1y_3 \end{cases} \quad y_1(0) = 1, y_2(0) = 1, y_3(0) = 1,$$

2. Suppose the population in California move outside of the state at a rate of 8% per year and that people from outside the state move in at a rate of 4% per year. Assuming the population inside and outside the state remain constant, what is the distribution of the population in California  $k$  years later? What is the distribution of the population as  $k$  becomes infinite?

Let the population at time  $t = 0$  inside and outside be  $z_0$  and  $y_0$  respectively.

- a. Write a set of equations for the population inside and outside California after one year. And convert the equation in part a into a matrix equation.
- b. Write a matrix equation for the distribution of the population after  $k$  years using the initial population.  $(y_0, z_0)$
- c. Find a matrix  $P$  that diagonalizes the matrix in part c.
- d. Find an expression for  $(y_k, z_k)$
- e. Discuss  $\lim_{k \rightarrow \infty} (y_k, z_k)$