

Give complete solutions to the following problems. Be sure to provide all the necessary steps to support your answers.

1. Use vector projection to find the closest vector to \mathbf{u} on the space \mathbf{W} spanned by the set S , then use the projection to find the distance from \mathbf{u} to the space \mathbf{W} , the error vector in the approximation. Use a picture of \mathbf{W} and \mathbf{u} to illustrate.

$$\mathbf{u} = (1, 2, 1), \quad S = \{(1, -1, 1), (1, 1, 2)\}$$

2. Consider the system

$$\begin{cases} x + y = 7 \\ -x + y = 0 \\ -x + 2y = -7 \end{cases}$$

- a. Find the associated normal system.
- b. Find the least squares solution.
- c. Find the error vector and the least squares error.

3. Find the standard matrix for the orthogonal projection onto the space \mathbf{W} spanned by the set S , then use the matrix to find the projection of the vector \mathbf{u} onto \mathbf{W} .

$$S = \{(2, 1, 1, 1), (1, 0, 1, 1), (-2, -2, 0, -1)\}, \quad \mathbf{u} = (2, 3, 9, 6).$$

4. Use the Gram-Schmidt process to find an orthonormal basis for \mathbf{W} in problem 3, then use the ONB to find the projection of \mathbf{u} onto \mathbf{W}

5. Consider the set of points.

X	-2	1	2	3	3
Y	-3	2	3	4	6

- a. Use a projection matrix with a linear independent basis for the column space to find the least squares solution
- b. Repeat using a matrix with an orthonormal basis for the column space to find the least squares solution.