Math002B Final Exam Review Spring 2012

Give complete solutions to the following problems be sure to provide all the necessary steps to support your answers.

1. Consider the linear system $\begin{cases} x_1 + x_2 + 2x_3 = 5 \\ x_1 + x_2 - 2x_3 = 1 \\ 3x_1 + 2x_2 - 4x_3 = 3 \end{cases}$

a) Convert the system of equations into a matrix equation Ax = b

b) Factor A into a product LU.

b) Use backward and forward substitution to find a solution to the system.

c) Find a basis for the column space of A in part a.

d) Find a basis for the Null space of A in part a.

e) Find a basis for the Null space of $\mathbf{A}^T$ in part a.

2. Find Bases for the Eigen-space of the matrix $\mathbf{A} = \begin{bmatrix} -1 & -2 & 0 \\ 1 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix}$

3 Find a basis for the orthogonal complement of the subspace of R^4 spanned by $\mathbf{v}_1 = (1,1,1,1); \mathbf{v}_2 = (1,2,0,1)$

4. Find the least squares solution of the linear system
$\begin{cases} x_1 + x_2 = 2 \\ -x_1 + x_2 = 0 \\ 3x_1 + x_2 = 5 \end{cases}$

5. Find the orthogonal projection of vector of b onto the column space of A.
$\mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}, \ \mathbf{A} = \begin{bmatrix} 1 & -1 \\ 1 & -2 \\ 1 & 1 \end{bmatrix}$

6. Consider the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where $\mathbf{v}_1 = (1,0,1); \mathbf{v}_2 = (1,1,0), \mathbf{v}_3 = (1,0,0)$

a) show that S form a basis for R^3

b) Let $T : R^3 \to R^3$ be a linear, find $T(2,1,2)$, given that
$T(\mathbf{v}_1) = (2,-1,4), \ T(\mathbf{v}_2) = (3,0,1), \ T(\mathbf{v}_3) = (-1,5,1),$

7. Find the Projection of the vector $\overrightarrow{AB}$ onto the plane $2x - y + z = 2$ where A(1,1,1) is a point on the plane and B(3,2,2) is a point off the plane, then use the projection to find the distance from the B to the plane.

8 Consider the basis $B=\{\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3\}$, and $B'=\{\mathbf{u}'_1,\mathbf{u}'_2,\mathbf{u}'_3\}$ for R^3, where

$$\mathbf{u}_1=\begin{bmatrix}-3\\0\\-3\end{bmatrix},\ \mathbf{u}_2=\begin{bmatrix}-3\\2\\-1\end{bmatrix},\ \mathbf{u}_3=\begin{bmatrix}1\\6\\-1\end{bmatrix},\mathbf{u}'_1=\begin{bmatrix}-6\\-6\\0\end{bmatrix},\ \mathbf{u}'_2=\begin{bmatrix}-2\\6\\4\end{bmatrix},\ \mathbf{u}'_3=\begin{bmatrix}-2\\-3\\7\end{bmatrix}$$

a) find the transition matrix from B' to B.

b) find the transition matrix from B to B'.

c) find $[\mathbf{w}]_B=(3,-1,2)$, find $[\mathbf{w}]_{B'}$

9. Given $T(u_1,u_2,u_3)=(u_1-u_2,u_2+u_3,u_3)$, where $\mathbf{u}=(u_1,u_2,u_3)$ in R^3

a) show that T is a linear operator.

d) a basis for the Kernel of T is

10. Consider $S=\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3\}$ where $\mathbf{v}_1(1,2,1,1)$, $\mathbf{v}_2(1,0,1,0)$, $\mathbf{v}_3(1,0,0,1)$,

a) Find the coordinate vector of $\mathbf{u}=(2,-1,-1,1)$ relative to S

b) Find the projection of $\mathbf{u}=(1,2,2,0)$ onto $span\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3\}$

11. Solve

$$\begin{cases}\dfrac{dy_1}{dt}=0.2y_1-0.1y_2\\[2mm] \dfrac{dy_2}{dt}=-0.1y_1+0.1y_2\end{cases},$$

$y_1(0)=100,\ y_2(0)=100$

12. Use an appropriate change of variable to transform the equation below to one free of cross product terms, the. What type of surface does the equation represents?

$$2x_1^2-3x_2^2+4x_3^2-2x_1x_2+4x_1x_3+6x_2x_3=36$$

13. Given $S=\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,\}=\{(1,0,1),(1,1,0),(0,1,1)\}$

a) Show that S forms a basis for R^3.

b) Transform S into an orthonormal basis for R^3.

b) write $\mathbf{w}=(2,1,2)$ as a linear combination of the orthonormal basis from part a.

d) Let $A=\begin{bmatrix}1&1&0\\0&1&1\\1&0&1\end{bmatrix}$, find matrices Q and R such that A = QR