Math002B Final Exam Review Winter 2013

Give complete solutions to the following problems be sure to provide all the necessary steps to support your answers.

1. Consider the linear system $\begin{cases} x_1 + x_2 + 2x_3 = 5 \\ x_1 + x_2 - 2x_3 = 1 \\ 3x_1 + 2x_2 - 4x_3 = 3 \end{cases}$

a) Convert the system of equations into a matrix equation Ax = b

b) Factor A into a product LU.

b) Use backward and forward substitution to find a solution to the system.

c) Find a basis for the column space of A in part a.

d) Find a basis for the Null space of A in part a.

e) Find a basis for the Null space of $\mathbf{A}^T$ in part a.

2. Find Bases for the Eigen-space of the matrix $\mathbf{A} = \begin{bmatrix} -1 & -2 & 0 \\ 1 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix}$

3 Find a basis for the orthogonal complement of the subspace of R^4 spanned by $\mathbf{v}_1 = (1,1,1,1); \mathbf{v}_2 = (1,2,0,1)$

4. Find the least squares solution of the linear system

$$\begin{cases} x_1 + x_2 = 2 \\ -x_1 + x_2 = 0 \\ 3x_1 + x_2 = 5 \end{cases}$$

5. Find the orthogonal projection of vector of b onto the column space of A.

$$\mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 1 & -1 \\ 1 & -2 \\ 1 & 1 \end{bmatrix}$$

6. Consider the set $S = \{\mathbf{v}_1 = (1,0,1); \mathbf{v}_2 = (1,1,0), \mathbf{v}_3 = (1,0,0)\}$
And $T: R^3 \to R^3$ A be a linear operator such that
$T(\mathbf{v}_1) = (2,-1,4), \ T(\mathbf{v}_2) = (3,0,1), \ T(\mathbf{v}_3) = (5,-1,5),$

a) show that S form a basis for R^3
b) Find the matrix [T]
c) Find the dimension of the null space of T

7. Find the Projection of the vector $\vec{AB}$ onto the plane $2x - y + z = 2$ where A(1,1,1) is a point on the plane and B(3,2,2) is a point off the plane, then use the projection to find the distance from the B to the plane.

8 Consider the basis

$$B = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}, \text{ and } B' = \{\mathbf{u}'_1, \mathbf{u}'_2, \mathbf{u}'_3\} \text{ for } R^3, \text{ where}$$

$$\mathbf{u}_1 = \begin{bmatrix} -3 \\ 0 \\ -3 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}, \mathbf{u}_3 = \begin{bmatrix} 1 \\ 6 \\ -1 \end{bmatrix}, \mathbf{u}'_1 = \begin{bmatrix} -6 \\ -6 \\ 0 \end{bmatrix}, \mathbf{u}'_2 = \begin{bmatrix} -2 \\ 6 \\ 4 \end{bmatrix}, \mathbf{u}'_3 = \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}$$

a) find the transition matrix from B' to B.

b) find the transition matrix from B to B'.

c) find $[\mathbf{w}]_B = (3, -1, 2)$, find $[\mathbf{w}]_{B'}$

9. Given $T(u_1, u_2, u_3) = (u_1 - u_2, u_2 + u_3, u_3)$, where $\mathbf{u} = (u_1, u_2, u_3)$ in R^3

a) show that T is a linear operator.

b) Find the associated matrix [T]

c) Find a basis for the Kernel of T is

10. Consider $S = \{\mathbf{v}_1(1,2,1,1),\ \mathbf{v}_2(1,0,1,0),\ \mathbf{v}_3(1,0,0,1)\}$

a) Find the coordinate vector of $\mathbf{u} = (2, -1, -1, 1)$ relative to S

b) Find the projection of $\mathbf{u} = (1, 2, 2, 0)$ onto $span\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$

11. Solve

$$\begin{cases} \dfrac{dy_1}{dt} = 0.2y_1 - 0.1y_2 \\ \dfrac{dy_2}{dt} = -0.1y_1 + 0.1y_2 \end{cases},$$

$y_1(0) = 100,\ y_2(0) = 100$

12. Transform the vectors $\mathbf{v}_1 = (1,1,1); \mathbf{v}_2 = (1,1,0), \mathbf{v}_3 = (1,0,0)$ into an orthonormal basis for R^3 , then express the vec $\mathbf{w} = (1,1,1)$ as a linear combination of the orthonormal basis.

13. Use an appropriate change of variable to transform the equation below to one free of cross product terms, the. What type of surface does the equation represents?

$$2x_1^2 - 3x_2^2 + 4x_3^2 - 2x_1x_2 + 4x_1x_3 + 6x_2x_3 = 36$$

14. Given $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3,\} = \{(1,0,1),(1,1,0),(0,1,1)\}$

a) Show that S forms a basis for R^3.

b) Transform S into an orthonormal basis for R^3.

c) write $\mathbf{w} = (2,1,2)$ as a linear combination of the orthonormal basis from part a.

d) Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, find matrices Q and R such that A = QR

15. Find the spectral decomposition of $\mathbf{A} = \begin{bmatrix} 6 & -2 \\ -2 & 3 \end{bmatrix}$ and give a geometric interpretation of the spectral decomposition of A.

16. Find a 3x3 matrix whose eigenvalues $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3$ and the corresponding eigenvector are $u_1 = (0,1,0), u_1 = (1,0,-1), u_1 = (1,0,1)$,