

Give complete solutions to the following problems be sure to provide all the necessary steps to support your answers.

1. Give example of the following
  - a) 3 x 3 system of equation that have a unique solution. Find the solutions using an rref matrix.
  - b) Modify the system of equations in part a so that the new system has infinite number of solutions. Give the rref form of the augmented matrix.
  - c) Modify the system in part a so that the system has no solution. Give the rref form of the augmented matrix.

2. Which of the matrices below are in row echelon form or row reduced echelon form.

a)  $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$     b)  $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 2 \end{bmatrix}$     c)  $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$     d)  $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Ans: REF:  
RREF:  
NEITHER:

3. Solve the system of equations using Gaussian elimination.

$$\begin{cases} x - y + z = 5 \\ x + y + z = 7 \\ 2x - 4y + 2z = 8 \end{cases}$$

4. Find a system of equations that corresponds to the given augmented matrix

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 1 & 2 & 1 & 0 \end{bmatrix}$$

5. The matrix below is the ref form of an augmented matrix.
  - a) Determine if the system is consistent or inconsistent, dependent or independent.
  - b) If the system is consistent give the solution.

$$\begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Consider the system of equations

$$\begin{cases} x + y + z = -1 \\ 2x - y + z = -2 \\ 3x - y + z = -1 \end{cases}$$

6. Give the elementary matrices that transform the associated coefficient matrix to the system below to an upper triangular matrix. This is matrix U.
7. Use the elementary matrices to find a lower triangular matrix L, so that  $A = LU$ .
8. Use Forward substitution to find solution to the system  $LC = b$ .
9. Use Backward substitution to find solutions to the given system. This is  $Ux = C$ .
10. Use Cramer's Rule to solve the system.

11 - 14. Consider the linear system.

$$\begin{cases} x + y + z = 4 \\ x + 2y + z = 6 \\ x = 1 \end{cases}$$

11. Show that the coefficient matrix associated with the system above is invertible. Call it A.
12. Find the Adjoint of matrix A:  $\text{Adj}(A)$
13. Use the  $\text{Adj}(A)$  to find the inverse matrix of A.
14. Solve the system of equations using the inverse matrix of A.

15. # 5 or number 9 in Section 1.8 Exercises.