Give complete solutions to the following problems be sure to provide all the necessary steps to support your answers.

1. Find the a basis for the Kernel of the given transformation defined by
$T(x_1, x_2, x_3) = (x_1 + x_2, -x_1 + x_2, 2x_2 + x_3)$

2. Determine if $T(x, y, z) = (3x - 4y, 2x - 5z)$ is a linear operator.

3. Let $u = (2,1,3)$ use matrix multiplication to find

   a) the reflection of $\boldsymbol{u}$ about the yz plane.
   b) the projection of $\boldsymbol{u}$ on the yz plane.

4. Let $T(x_1, x_2, x_3) = (x_1 + 2x_2, x_1 - x_3)$ be a linear transformation

   a) Find the domain and the co-domain of T
   b) Find the standard matrix for linear operator

5. Given $T(1,0,0) = (1,1,1), T(0,1,0) = (-1,0,1), T(0,0,1) = (1,1,2)$
find an expression for T and use it to find T(3,2,2)

6. Find the projection of $\mathbf{w} = (2,1,2)$ onto $\mathbf{u} \times \mathbf{v}$ given $\mathbf{v} = (3,1,1), \mathbf{u} = (-1,2,3)$

7. Find the standard matrix for the composition of a rotation by 30 degrees about the x axis, followed by a rotation of 30 degrees about the z axis, followed by a contraction by a factor k =1/2 for vectors in 3D space.

8. Prove that the given linear operator defined below is one to one, then Find $T^{-1}(1,1,1)$

$$
\begin{aligned}
w_1 &= x_1 + 2x_2 + x_3 \\
w_2 &= -2x_1 + x_2 + 4x_3 \\
w_3 &= 7x_1 + 4x_2 - 5x_3
\end{aligned}
$$

9. Solve the system of differential equation

$$
\begin{cases}
\dfrac{dy_1}{dt} = y_1 + y_2 + y_3 \\
\dfrac{dy_2}{dt} = \quad 2y_2 - y_3, \qquad y_1(0) = 5,\ y_2(0) = 5,\ y_3(0) = 5, \\
\dfrac{dy_3}{dt} = \qquad -2y_3
\end{cases}
$$

10. Given $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3,\} = \{(0,1,0), (1,1,1), (0,1,1)\}$

   a) Show that S forms a basis for R^3.
   b) Transform S into an orthonormal basis for R^3.