Give complete solutions to the following problems be sure to provide all the necessary steps to support your answers.

1. Find the a basis for the Kernel of the given transformation defined by $T(x_1, x_2, x_3) = (x_1 + x_2, -x_1 + x_2, 2x_2 + x_3)$

2. Determine if $T(x, y, z) = (3x - 4y, 2x - 5z)$ is a linear operator.

3. Let $u = (2,1,3)$ use matrix multiplication to find
   a) the reflection of $\boldsymbol{u}$ about the yz plane.
   b) the projection of $\boldsymbol{u}$ on the yz plane.

4. Let $T(x_1, x_2, x_3) = (x_1 + 2x_2, x_1 - x_3)$ be a linear transformation
   a) Find the domain and the co-domain of T
   b) Find the standard matrix for linear operator

5. Given $T(1,0,0) = (1,1,1), T(0,1,0) = (-1,0,1), T(0,0,1) = (1,1,2)$
   find an expression for T and use it to find T(3,2,2)

6. Find the projection of $\mathbf{w} = (2,1,2)$ onto $\mathbf{u} \times \mathbf{v}$ given $\mathbf{v} = (3,1,1), \mathbf{u} = (-1,2,3)$

7. Find the standard matrix for the composition of a rotation by 30 degrees about the x axis, followed by a rotation of 30 degrees about the z axis, followed by a contraction by a factor k =1/2 for vectors in 3D space.

8. Prove that the given linear operator defined below is one to one, then Find $T^{-1}(1,1,1)$

$$w_1 = x_1 + 2x_2 + x_3$$
$$w_2 = -2x_1 + x_2 + 4x_3$$
$$w_3 = 7x_1 + 4x_2 - 6x_3$$

10. Find the orthogonal projection of vector of b onto the column space of A.

$$\mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 1 & -1 \\ 2 & -2 \\ 1 & 1 \end{bmatrix}$$

11. Find the least squares solution of the linear system $\begin{cases} x_1 - x_2 = 0 \\ x_1 - 2x_2 = -10 \\ 2x_1 - x_2 = 6 \end{cases}$

12. Solve the system of differential equation

$$\begin{cases} \dfrac{dy_1}{dt} = y_1 + y_2 + y_3 \\ \dfrac{dy_2}{dt} = 2y_2 - y_3, \qquad y_1(0) = 5,\ y_2(0) = 5,\ y_3(0) = 5, \\ \dfrac{dy_3}{dt} = -2y_3 \end{cases}$$

13. Given $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\} = \{(0,1,0), (1,1,1), (0,1,1)\}$
   a) Show that S forms a basis for R^3.
   b) Transform S into an orthonormal basis for R^3.

14) Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, find matrices Q and R such that A = QR