

Give complete solutions to the following problems be sure to provide all the necessary steps to support your answers.

Given the two linear systems

$$1. \begin{cases} 2x + y + z = 2 \\ x - y + z = 1 \end{cases} \quad 2. \begin{cases} 2x + y + z = 2 \\ x - y + z = 1 \\ 4x - y + 3z = 6 \end{cases}$$

Use Gaussian eliminations to show if any of the systems above is consistent, in the case the system is consistent, give all solutions

3, & 4. Give the elementary matrix that performs the following row operation in a 3x3 system of linear equations

3. Adds twice equation 3 to equation 2.

4. Interchange equations 1 and 2.

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 \\ 0 & 4 \\ 3 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix}$$

Compute the following (where possible)

5. $2AB$

6. AB^T

7. $(BC)D$

8. $\text{trace}(A)$

9. Give a 3x3 system of equations that has infinitely many solutions, explain the method you use to construct the system.

Problems 10- 16. Given

$$\begin{cases} 2x + y = 3 \\ 2x - y = 1 \\ 6x - y = 5 \end{cases}$$

10. Give the augmented matrix
11. Give the coefficient matrix
12. Give the associated matrix equation
13. Use row operations to put the system into row echelon form. Be sure to write all row operations applied to the system/augmented matrix.
14. State if the system is consistent or inconsistent.
15. In case, there is/are solutions, give all solutions to the system.
16. State if the system is dependent or independent.
17. Give conditions on b_1 , and b_2 so that the given system has a unique solution.

$$\begin{cases} x + y = b_1 \\ x + 2y = b_2 \end{cases}$$