**Math02B Test 2 Review Spring 2012** Name_____

Give complete solutions to the following problems be sure to provide all the necessary steps to support your answers.

1. Prove or disapprove (use theorems/results from text and lecture)

   a) $(1,3),(2,1),$ and $(5,1)$ form a basis for $R^2$

   b) $(1,2,1),(2,1,2),(1,2,3)$ and $(3,2,1)$ span $R^3$

2. Find the rank an nullity of the matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 2 \\ 1 & 1 & 0 \end{bmatrix}$

3. Find the coordinates of vector w relative to the set S.
   $w = (2,-1,3),\ S = \{(3,3,3),(1,0,0),(2,2,0)\}$

4. Determine the dimension of and a basis for the solution space of the system $\begin{cases} 2x_1 + x_2 + 3x_3 = 0 \\ x_1 \quad\quad + 5x_3 = 0 \\ \quad\quad x_2 + x_3 = 0 \end{cases}$

5. Assume the given set of vectors below are vectors in 3-space with their initial points at the origin, determine if each given set of vectors lie on the same plane, or the same line. Prove your answer.

   a) $v_1 = (-6,7,2), v_2 = (3,2,4),$ *and* $v_3 = (4,-1,2).$

   b) $v_1 = (2,-1,4), v_2 = (4,2,3),$ *and* $v_3 = (2,7,-6).$

6. Use a linear system of equations and Gaussian elimination to show that the given position vectors span $\Re^3$ .
   $\mathbf{v}_1 = (1,1,1), \mathbf{v}_2 = (1,2,1), \mathbf{v}_3 = (2,1,3),$ and $\mathbf{v}_4 = (5,3,8)$

7. Let W be the set of all solutions to the equation $x - 2y + z = 0$ Determine if W is a subspace of $R^3$. Use definitions and theorems to prove your answer.

8. For the matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$ ,determine whether the solution space Ax=0 is a lin through the origin, a plane through the origin, or only the origin, then give the parametric equation of the line or the plane in case the solution is not the zero vector, or the equation of the plane in rectangular coordinates in the event the solution space is a plane.

9. Find bases for the column space, the row space, and the null space of the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$$

10. Determine if the given vector b is in the column space of **A**, and if so, express b as linear combination of the column vectors of **A** .

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 1 \\ 9 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}$$

11- 15 Given $P(2,3,2)$, $Q(5,8,6)$, $O(0,0,0)$

11. Find $\mathbf{Proj}_{\vec{OQ}} \vec{OP}$ Ans_____

12. Find a unit vector in the direction of $\vec{OP}$ Ans_____

13. Find the angle between $\overrightarrow{OP}$ and $\overrightarrow{QP}$ Ans_____

14. Find the distance from $p(3,2,1)$ to the plane Ans_____
$2x+y+z=1$

15. Find the distance from $p(3,2,1)$ to the line Ans_____
$x=1-2, y=2+t, z=t$