

Give complete solutions to the following problems be sure to provide all the necessary steps to support your answers.

- Find the a basis for the Kernel of the given transformation defined by

$$T(x_1, x_2, x_3) = (x_1 + x_2, -x_1 + x_2, 2x_2 + x_3)$$
- Determine if $T(x, y, z) = (3x - 4y, 2x - 5z)$ is a linear operator.
- Let $\mathbf{u} = (2, 1, 3)$ use matrix multiplication to find
 - the reflection of \mathbf{u} about the x axis.
 - the projection of \mathbf{u} on the yz plane.
- Let $T(x_1, x_2) = (x_1 + 2x_2, x_1 - x_2)$ be a linear transformation
 - Find the domain and the co-domain of T
 - Find the standard matrix for linear operator
- Given $T(1, 0, 0) = (1, 1, 1), T(0, 1, 0) = (-1, 0, 1), T(0, 0, 1) = (1, 1, 2)$
find an expression for T and use it to find $T(3, 2, 2)$
- Find the projection of $\mathbf{w} = (2, 1, 2)$ onto $\mathbf{u} \times \mathbf{v}$; given $\mathbf{v} = (3, 1, 1), \mathbf{u} = (-1, 2, 3)$
- Find the standard matrix for the composition of a rotation by 30 degrees about the x axis, followed by a rotation of 30 degrees about the z axis, followed by a contraction by a factor $k = 1/2$ for vectors in 3D space.
- Prove that the given linear operator defined below is one to one, then Find $T^{-1}(1, 1, 1)$

$$w_1 = x_1 + 2x_2 + x_3$$

$$w_2 = -2x_1 + x_2 + 4x_3$$

$$w_3 = 7x_1 + 4x_2 - 5x_3$$
- Solve the system of differential equation

$$\begin{cases} \frac{dy_1}{dt} = y_1 + y_2 + y_3 \\ \frac{dy_2}{dt} = 2y_2 - y_3, & y_1(0) = 5, y_2(0) = 5, y_3(0) = 5, \\ \frac{dy_3}{dt} = -2y_3 \end{cases}$$
- Given $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\} = \{(0, 1, 0), (1, 1, 1), (0, 1, 1)\}$
 - Show that S forms a basis for \mathbb{R}^3 .
 - Transform S into an orthonormal basis for \mathbb{R}^3 .